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Non-uniform double slot suction (injection) into water boundary layer flows over a cylinder

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ABSTRACT

An analysis is performed to study the effect of non-uniform double slot suction (injection) into a steady two-dimensional laminar boundary layer flow when fluid properties such as viscosity and Prandtl number are inverse linear functions of temperature. Non-similar solutions have been obtained from the starting point of the streamwise co-ordinate to the exact point of separation. By applying an implicit finite difference scheme in combination with the quasi-linearization technique and an appropriate selection of the finer step sizes along the streamwise direction, the difficulties arising at the starting point of the streamwise co-ordinate, at the edges of the slot and at the point of separation have been overcome. The results indicate that the separation can be delayed by non-uniform double slot suction and also by moving the slot downstream. However, the effect of non-uniform double slot injection is just the opposite.

1. Introduction

Recently, a wide range of boundary layer flow problems have been studied that took non-similarity into account. These studies are of practical importance when non-similarity arises due to the free stream velocity, the curvature of the body, the surface mass transfer, or a combined effect of all these factors. A review of non-similarity solution methods along with citations of some relevant publications is found in Dewey and Gross [1]. Since then, there have been several studies pertaining to non-similar flows by the finite difference method [2,3] and the implicit finite difference method in combination with quasi-linearization technique [4,5].

Fluid viscosity and thermal conductivity are the main governing fluid properties in laminar water boundary layer forced flow, and hence their variations also can be expected to affect separation of the boundary layer from the solid surface. As these properties are temperature dependent, variations are most easily accomplished in the boundary layer by maintaining a temperature difference between the solid wall and the fluid. In practice, wall heating has been shown to be an efficient way to stabilize boundary layer flow and to delay the flow transition when fluid viscosity decreases by heating. Indeed for undersea applications, surface heating is an effective means of controlling boundary layer separation since heating promotes stability through the interplay among the thermal boundary layer, the temperature dependent viscosity, and momentum balance in the crucial region near the wall. Several investigators [6–8] have studied steady laminar boundary layer flows over various heated bodies with variable fluid properties.

Mass transfer through a wall slot (i.e., mass transfer occurring in a small porous section of the body surface while no mass transfer is occurring in the remaining part of it) into the boundary layer is of interest for various potential applications including thermal protection, energizing the inner portion of the boundary layer in adverse pressure gradient, and skin friction reduction on control surfaces. Moreover, mass transfer through a slot strongly influences the development of a boundary layer along a surface and in particular can prevent or at least delay separation of the viscous region.

Different studies [9,10] show the effect of single slot injection (suction) into steady water and compressible boundary layer flows over two-dimensional and axi-symmetric bodies. Moreover, Roy [11] and Subhashini et al. [12] have investigated the influence of non-uniform double slot injection (suction) on compressible boundary layer flows over cylinder, sphere and yawed cylinder, respectively. Also, in more recent studies, Roy et al. [13], and Roy and Saikrishnan [14] have reported the influence of non-uniform double slot injection (suction) on an incompressible boundary layer flow over a slender cylinder and within a diverging channel, respectively. Therefore, as a step towards the eventual development of the study of mass transfer into the boundary layer flows, it is interesting as well as useful to investigate the influence of non-uniform double slot injection (suction) on steady laminar non-similar water boundary layer flow over a cylinder.

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Nomenclature

A	mass transfer parameter
Ec	Eckert number, viscous dissipation parameter
f	dimensionless stream function
F	dimensionless velocity along surface
G	dimensionless temperature
L	characteristic length
N	viscosity ratio
Pr	Prandtl number
R	radius of the cylinder
Re_L	Reynolds number
T	dimensional temperature
u, v	velocity components along x- and y-directions, respectively
$u_e(x)$	potential flow velocity
$v_w(x)$	surface mass transfer distribution
w^*	slot length parameter
x, y	cartesian co-ordinates along and normal to surface, respectively
$\bar{x}_0, \bar{x}_1$	slot location parameters

Greek symbols

β	pressure gradient parameter
η	transformed variable
μ	dynamic viscosity
ξ	a scaled streamwise co-ordinate
ψ	dimensional stream function

Subscripts

∞	conditions in the free stream
e, w	denote conditions at the edge of the boundary layer and on the surface, respectively
η, ξ	partial derivatives with respect to η and ξ

The present investigation considers the effect of non-uniform double slot suction (injection) on the steady non-similar water boundary layer flow over a cylinder with variable viscosity and Prandtl number. This analysis is useful in understanding many boundary layer problems of practical importance for undersea applications, for example, in suppressing recirculating bubbles and controlling transition and/or separation of the boundary layer over submerged bodies. The non-similar solutions have been obtained starting from the origin of the streamwise co-ordinate to the point of separation (zero skin friction in the streamwise direction) using a quasi-linearization technique with an implicit finite difference scheme. There are two free parameters in this problem, one measuring the length of the slot (i.e., the part of the body surface in which there is a mass transfer) and the other fixing the position of the slot. Thus, these two parameters permit variations in the slot length and movement of slot position.

2. Mathematical formulation

We consider laminar non-similar boundary layer forced convection flow (of water) with temperature-dependent viscosity and Prandtl number over a cylinder when the free stream velocity and the non-uniform mass transfer (suction/injection in a slot) vary with the axial distance (x) along the surface. Let x and y be the curvilinear co-ordinates along and perpendicular to the body surface, respectively, with u and v being the corresponding velocity components (see Fig. 1). The fluid is assumed to flow with moderate velocity, and the temperature difference between the wall and the free stream is small (<40 °C). In this range of temperature (i.e., 0–40 °C), variation of both density (ρ) and specific heat (C_p) is less than 1% (see Table 1), therefore they are considered to be constants. However, since the variation of thermal conductivity (k) and viscosity (μ) [and hence Prandtl number (Pr)] with temperature is quite significant, the viscosity and Prandtl number are assumed to vary as an inverse linear function of temperature (T) [4]:

$$\mu = \frac{1}{b_1 + b_2 T} \quad \text{and} \quad Pr = \frac{1}{c_1 + c_2 T}, \tag{1}$$

where

$$b_1 = 53.41, \quad b_2 = 2.43, \quad c_1 = 0.068 \quad \text{and} \quad c_2 = 0.004. \tag{2}$$

The numerical data used for these correlations are taken from [15]. The relations (1) and (2) are reasonably good approximations for liquids such as water, particularly for small wall and ambient temperature differences. As the fluid is incompressible, the contribution of heating due to compression is very small and it has been neglected. The effect of viscous dissipation is included in the analysis. The fluid at the edge of the boundary layer is maintained at a constant temperature T_∞, and the body has a uniform temperature T_w. The blowing rate of the fluid is assumed to be small and it does not affect the inviscid flow at the edge of the boundary layer. It is also assumed that the injected fluid possesses the same physical properties as the boundary layer fluid and has a static temperature equal to the wall temperature. The boundary layer equations governing the flow are given by [9,16]:

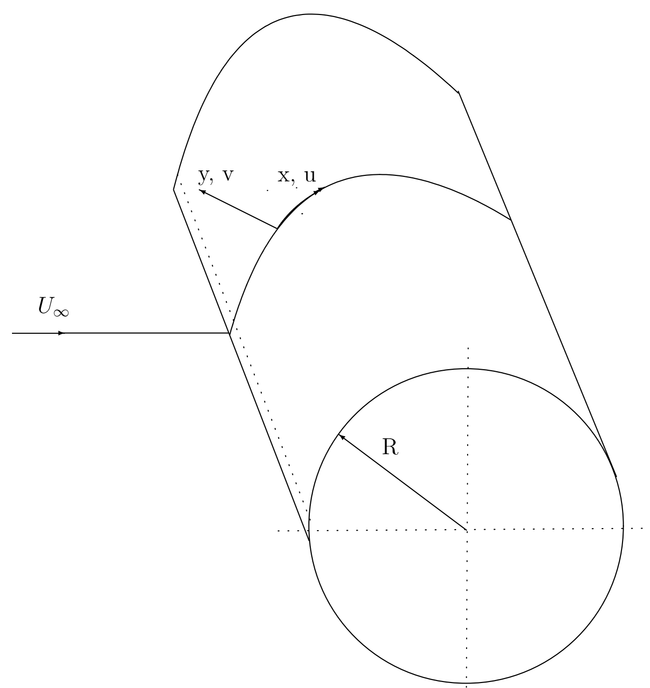

Fig. 1. Flow geometry and co-ordinate system.

$$u_x + v_y = 0, \tag{3}$$

$$uu_x + vu_y = u_e(u_e)_x + \rho^{-1}(\mu u_y)_y, \tag{4}$$

$$uT_x + vT_y = \rho^{-1}\left(\frac{\mu}{Pr}T_y\right)_y + \frac{\mu}{\rho C_p}\left(u_y^2\right), \tag{5}$$

Table 1
Values of thermo-physical properties of water at different temperatures [15]

Temperature (T) (°C)	Density (ρ) (g/cm^3)	Specific heat (C_p) (J × 10^7/kg K)	Thermal conductivity (k) (erg × 10^5/cm s K)	Viscosity (μ) (g × 10^{-2}/cm s)	Prandtl number, Pr
0	1.00228	4.2176	0.5610	1.7930	13.48
10	0.99970	4.1921	0.5800	1.3070	9.45
20	0.99821	4.1818	0.5984	1.0060	7.03
30	0.99565	4.1784	0.6154	0.7977	5.12
40	0.99222	4.1785	0.6305	0.6532	4.32
50	0.98803	4.1806	0.6435	0.5470	3.55

with boundary conditions

$$u(x,0)=0,\quad v(x,0)=v_{\rm w}(x),\quad T(x,0)=T_{\rm w}=\text{constant},$$
$$u(x,\infty)=u_{\rm e}(x),\quad T(x,\infty)=T_\infty=\text{constant}. \tag{6}$$

Applying the following transformations:

$$\xi=\int_0^x\left(\frac{u_{\rm e}}{u_\infty}\right){\rm d}\left(\frac{x}{L}\right),\quad \eta=\left(\frac{Re_{\rm L}}{2\xi}\right)^{1/2}\left(\frac{u_{\rm e}}{u_\infty}\right)\left(\frac{y}{L}\right),$$
$$\psi(x,y)=u_\infty L\left(\frac{2\xi}{Re_{\rm L}}\right)^{1/2}f(\xi,\eta),\quad u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}, \tag{7}$$
$$Re_{\rm L}=\frac{u_\infty L\rho}{\mu},\quad G=\frac{T-T_\infty}{T_{\rm w}-T_\infty},\quad F=f_\eta(\xi,\eta)=\frac{u}{u_{\rm e}},$$

to Eqs. (3)–(5), we see that the continuity Eq. (3) is identically satisfied, and Eqs. (4) and (5) reduce to a non-dimensional form, respectively, as:

$$(NF_\eta)_\eta+fF_\eta+\beta(\xi)(1-F^2)=2\xi(FF_\xi-f_\xi F_\eta), \tag{8}$$

$$(NPr^{-1}G_\eta)_\eta+fG_\eta+NEc\left(\frac{u_{\rm e}}{u_\infty}\right)^2F_\eta^2=2\xi(FG_\xi-f_\xi G_\eta), \tag{9}$$

where

$$N=\frac{\mu}{\mu_\infty}=\frac{b_1+b_2T_\infty}{b_1+b_2T}=\frac{1}{1+a_1G},\quad Pr=\frac{1}{c_1+c_2T}=\frac{1}{a_2+a_3G},$$
$$a_1=\left(\frac{b_2}{b_1+b_2T_\infty}\right)\Delta T_{\rm w},\quad a_2=c_1+c_2T_\infty,\quad a_3=c_2\Delta T_{\rm w},$$
$$\beta(\xi)=\frac{2\xi}{u_{\rm e}}\frac{{\rm d}u_{\rm e}}{{\rm d}\xi},\quad Ec=\frac{u_\infty^2}{C_p\Delta T_{\rm w}},\quad \Delta T_{\rm w}=(T_{\rm w}-T_\infty).$$

The transformed boundary conditions are

$$\begin{aligned}F(\xi,0)&=0,\quad G(\xi,0)=1,\\ F(\xi,\infty)&=1,\quad G(\xi,\infty)=0,\end{aligned} \tag{10}$$

where

$$f=\int_0^\eta F\,{\rm d}\eta+f_{\rm w}\ \text{ and }\ f_{\rm w}=-\xi^{-1/2}\left(\frac{Re_{\rm L}}{2}\right)^{1/2}\int_0^x\left(\frac{v_{\rm w}}{u_\infty}\right){\rm d}\left(\frac{x}{L}\right). \tag{11}$$

In the case of a circular cylinder of radius R, the velocity at the edge of the boundary layer and the surface mass transfer are functions of $\bar{x}$, which gives rise to non-similarity. The free stream velocity distribution for the case of a circular cylinder and the distance from the axis of the body are given by Schlichting [16]

$$\frac{u_{\rm e}}{u_\infty}=2\sin(\bar{x}),\quad \bar{x}=\frac{x}{R}.$$

The values of ξ, β and $f_{\rm w}$ are given by

$$\xi=2P_1,\quad \beta=2\cos(\bar{x})P_2^{-1}, \tag{12}$$

$$f_{\rm w}=\begin{cases}0, & \bar{x}\leqslant\bar{x}_0,\\ A(2P_1)^{-1/2}C(\bar{x},\bar{x}_0), & \bar{x}_0\leqslant\bar{x}\leqslant\bar{x}_0^*,\\ A(2P_1)^{-1/2}C(\bar{x}_0^*,\bar{x}_0), & \bar{x}_0^*\leqslant\bar{x}\leqslant\bar{x}_1,\\ A(2P_1)^{-1/2}\{C(\bar{x}_0^*,\bar{x}_0)+C(\bar{x},\bar{x}_1)\}, & \bar{x}_1\leqslant\bar{x}\leqslant\bar{x}_1^*,\\ A(2P_1)^{-1/2}\{C(\bar{x}_0^*,\bar{x}_0)+C(\bar{x}_1^*,\bar{x}_1)\}, & \bar{x}\geqslant\bar{x}_1^*,\end{cases} \tag{13}$$

where the function

$$C(\bar{x},\bar{x}_0)=1-\cos\{w^*(\bar{x}-\bar{x}_0)\}\quad\text{and}\quad P_1=1-\cos\bar{x},\quad P_2=1+\cos\bar{x}.$$

Here $v_{\rm w}$ is taken as

$$v_{\rm w}(\bar{x})=\begin{cases}0, & \bar{x}\leqslant\bar{x}_0,\\ -u_\infty\left(\frac{Re_{\rm L}}{2}\right)^{-1/2}Aw^*\sin\{w^*(\bar{x}-\bar{x}_0)\}, & \bar{x}_0\leqslant\bar{x}\leqslant\bar{x}_0^*,\\ 0, & \bar{x}_0^*\leqslant\bar{x}\leqslant\bar{x}_1,\\ -u_\infty\left(\frac{Re_{\rm L}}{2}\right)^{-1/2}Aw^*\sin\{w^*(\bar{x}-\bar{x}_1)\}, & \bar{x}_1\leqslant\bar{x}\leqslant\bar{x}_1^*,\\ 0, & \bar{x}\geqslant\bar{x}_1^*,\end{cases}$$

where w^*, $\bar{x}_0$ and $\bar{x}_1$ are the three free parameters which determine the slot length and slot location. The function $v_{\rm w}$ is continuous for all values of $\bar{x}$ and it has a non-zero value only in the interval $[\bar{x}_0,\bar{x}_0^*]$ and $[\bar{x}_1,\bar{x}_1^*]$. This type of function allows the mass transfer to change slowly in the neighbourhood of the leading and trailing edges of the slot. The parameter $A>0$ or $A<0$ indicates suction or injection, respectively.

It is convenient to express Eqs. (8) and (9) in terms of $\bar{x}$ instead of ξ. Eq. (12) gives the relation between ξ and $\bar{x}$ as

$$\xi\frac{\partial}{\partial\xi}=B(\bar{x})\frac{\partial}{\partial\bar{x}}, \tag{14}$$

where $B(\bar{x})=\tan\left(\frac{\bar{x}}{2}\right)$.

Substituting Eqs. (12) and (14) in Eqs. (8) and (9), we obtain

$$(NF_\eta)_\eta+fF_\eta+\beta(\bar{x})(1-F^2)=2B(\bar{x})(FF_{\bar{x}}-f_{\bar{x}}F_\eta), \tag{15}$$

$$(NPr^{-1}G_\eta)_\eta+fG_\eta+NEc\left(\frac{u_{\rm e}}{u_\infty}\right)^2F_\eta^2=2B(\bar{x})(FG_{\bar{x}}-f_{\bar{x}}G_\eta). \tag{16}$$

The boundary conditions reduce to

$$\begin{aligned}F(\bar{x},0)&=0,\quad G(\bar{x},0)=1,\\ F(\bar{x},\infty)&=1,\quad G(\bar{x},\infty)=0,\end{aligned} \tag{17}$$

where $f=\int_0^\eta F\,{\rm d}\eta+f_{\rm w}$ and $f_{\rm w}$ is given by Eq. (13). The skin friction coefficient at the wall can be expressed in the form

$$C_{\rm f}(Re_{\rm L})^{1/2}=4P_2P_1^{1/2}N_{\rm w}(F_\eta)_{\rm w}, \tag{18}$$

where $C_{\rm f}=\frac{2\tau_{\rm w}}{\rho u_\infty^2}=\frac{2\left(\mu\frac{\partial u}{\partial y}\right)_{\rm w}}{\rho u_\infty^2}$.

Similarly, the heat transfer coefficient can be written in terms of a Nusselt number as

$$Nu(Re_{\rm L})^{-1/2}=-2^{1/2}\cos(\bar{x}/2)(G_\eta)_{\rm w}, \tag{19}$$

where $Nu=-\frac{L\left(\frac{\partial T}{\partial y}\right)_{\rm w}}{\Delta T_{\rm w}}$.

3. Method of solution

The boundary value problem represented by Eqs. (15)–(17) is tackled by the implicit finite difference scheme in combination with the quasi-linearization technique. Quasi-linear technique can be viewed as a generalization of the Newton–Raphson approximation technique in functional space. An iterative sequence of linear equations is carefully constructed to approximate the non-linear equations (15) and (16) for achieving quadratic convergence and monotonicity. The quadratic convergence and monotonicity are unique characteristics for quasilinear implicit finite difference scheme which makes this scheme superior than built-in iteration of upwind or finite amplitude techniques.

Applying the quasi-linearization technique [5,9,17], we replace the non-linear partial differential equations (15) and (16) by an iterative sequence of linear equations as follows:

$$X_1^k F_{\eta\eta}^{k+1} + X_2^k F_\eta^{k+1} + X_3^k F^{k+1} + X_4^k F_{\bar{x}}^{k+1} + X_5^k G_\eta^{k+1} + X_6^k G^{k+1} = X_7^k, \quad (20)$$

$$Y_1^k G_{\eta\eta}^{k+1} + Y_2^k G_\eta^{k+1} + Y_3^k G^{k+1} + Y_4^k G_{\bar{x}}^{k+1} + Y_5^k F_\eta^{k+1} + Y_6^k F = Y_7^k, \quad (21)$$

where the coefficient functions with iterative index k are known and functions with iterative index $k+1$ are to be determined.

The boundary conditions become

$$F^{k+1} = 0, \quad G^{k+1} = 1 \quad \text{at } \eta = 0,$$
$$F^{k+1} = 1, \quad G^{k+1} = 0 \quad \text{at } \eta = \eta_\infty,$$

where η_∞ is the edge of the boundary layer. The coefficients in Eqs. (20) and (21) are given by

$$X_1^k = N,$$
$$X_2^k = -a_1 N^2 G_\eta + f + 2B(\bar{x}) f_{\bar{x}},$$
$$X_3^k = -2\beta F - 2B(\bar{x}) F_{\bar{x}},$$
$$X_4^k = -2B(\bar{x}) F,$$
$$X_5^k = -a_1 N^2 F_\eta,$$
$$X_6^k = -a_1 N^2 F_{\eta\eta} + 2a_1^2 N^3 F_\eta G_\eta,$$
$$X_7^k = -\beta(1+F^2) + (1-N^{-1}) N^2 F_{\eta\eta} - 2a_1 N^3 F_\eta G_\eta + a_1 N^2 F_\eta G_\eta - 2B(\bar{x}) F F_{\bar{x}},$$
$$Y_1^k = N Pr^{-1},$$
$$Y_2^k = 2a_3 N G_\eta - 2a_1 N^2 Pr^{-1} G_\eta + f + 2B(\bar{x}) f_{\bar{x}},$$
$$Y_3^k = -a_1 N^2 Pr^{-1} G_{\eta\eta} + a_3 N G_{\eta\eta} + 2a_1^2 N^3 Pr^{-1} G_\eta^2 - 2a_1 a_3 N^2 G_\eta^2 - a_1 Ec \left(\frac{u_e}{u_\infty}\right)^2 N^2 F_\eta^2,$$
$$Y_4^k = -2B(\bar{x}) F,$$
$$Y_5^k = 2Ec \left(\frac{u_e}{u_\infty}\right)^2 N F_\eta,$$
$$Y_6^k = -2B(\bar{x}) G_{\bar{x}},$$
$$Y_7^k = a_3 N G_{\eta\eta} G + (1-N^{-1}) N^2 Pr^{-1} G_{\eta\eta} + Ec \left(\frac{u_e}{u_\infty}\right)^2 N F_\eta^2 + Ec \left(\frac{u_e}{u_\infty}\right)^2 (1-N^{-1}) N^2 F_\eta^2 + a_1 N^2 Pr^{-1} G_\eta^2 - 2a_1 N^3 Pr^{-1} G_\eta^2 + 2a_3 N^2 G_\eta^2 - a_3 N G_\eta^2 - 2B(\bar{x}) F G_{\bar{x}}.$$

Now, the resulting sequence of linear partial differential equations (20) and (21) are expressed in difference form using a central difference scheme in η-direction and backward difference scheme in $\bar{x}$-direction. In each iteration step, the equations were then reduced to a system of linear algebraic equations with a block tri-diagonal structure which is solved using Varga's algorithm [18]. The step size in the η-direction has been chosen as $\Delta\eta = 0.01$ throughout the computations as it was found that a further decrease in $\Delta\eta$ does not change the results up to the fourth decimal place. In the $\bar{x}$-direction, $\Delta\bar{x} = 0.01$ was used for small values of $\bar{x} (\leqslant 0.5)$, then it was decreased to $\Delta\bar{x} = 0.005$. This value of $\Delta\bar{x}$ was used for $\bar{x} \leqslant 1.20$, thereafter the step size has been reduced further, ultimately choosing a value $\Delta\bar{x} = 0.0001$ near the point of zero skin friction. This was done because the convergence becomes slower when the point of vanishing meridional skin friction is approached. A convergence criterion based on the relative difference between the current and the previous iterations has been used. The solution is assumed to have converged and the iterative process is terminated when

$$\max\left\{\left|(F_\eta)_w^{k+1} - (F_\eta)_w^k\right|, \left|(G_\eta)_w^{k+1} - (G_\eta)_w^k\right|\right\} < 10^{-4}.$$

4. Results and discussion

Computations were carried out for various values A $(-0.5 \leqslant A \leqslant 0.5)$, Ec $(0 \leqslant Ec \leqslant 0.1)$ and with $\bar{x}_0 = 0.5$ and $\bar{x}_1 = 1.25$. Numerical results are displayed in graphs using MATLAB. In order to verify the correctness of the procedure, solutions have been obtained for the non-similar incompressible flows by substituting $Pr = 1.0$ to compare the skin friction and heat transfer parameters $[(F_\eta)_w, (G_\eta)_w]$ with the results of Venkatachala and Nath [3] by finite difference method. Comparison of the results $[C_f (Re_L)^{1/2}, Nu(Re_L)^{-1/2}]$ is also made with the steady state results of Eswara and Nath [4], who studied the problem of an unsteady non-similar two-dimensional and axi-symmetric water boundary layer with variable viscosity and Prandtl number. As the results are found to be in very good agreement, they are not presented here.

In Fig. 2, the effect of non-uniform suction of a single slot located at the position $\bar{x}_0 = 0.5$ on the skin friction is compared with that over a non-uniform suction of double slots situated at $\bar{x}_0 = 0.5$ and $\bar{x}_1 = 1.25$. It is observed that the separation is delayed and the point of separation moves further downstream with a double slot suction than with a single slot suction. Hence, double slot suction is more effective in delaying separation than single slot suction. In both the double and single slot suction cases, the skin friction gradually increases from the leading edge of the slot, attains a maximum, and then starts decreasing at the rear end of the slot.

The effect of mass transfer on skin friction, $C_f (Re_L)^{1/2}$, and the heat transfer coefficient, $Nu(Re_L)^{-1/2}$, in the case of non-uniform double slot suction is presented in Fig. 3. The skin friction and heat transfer coefficients increase with an increase in mass transfer rates, i.e., the value of A ($A > 0$). Also, the point of separation moves further downstream with the increase of A. The plot for skin friction and heat transfer coefficients in the case of non-uniform injection in a double slot is presented in Fig. 4. In contrast to the results

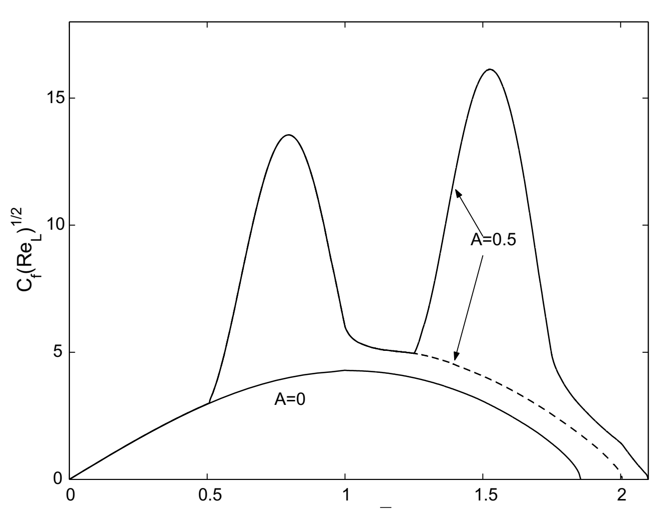

Fig. 2. Effect of single and double slot suctions on skin friction when $T_\infty = 18.7$, $T_w = 28.7$, $Ec = 0.05$ and $\omega^* = 2\pi$. (------) slot location at $\bar{x}_0 = 0.5$. (——) slot locations at $\bar{x}_0 = 0.5, \bar{x}_1 = 1.25$.

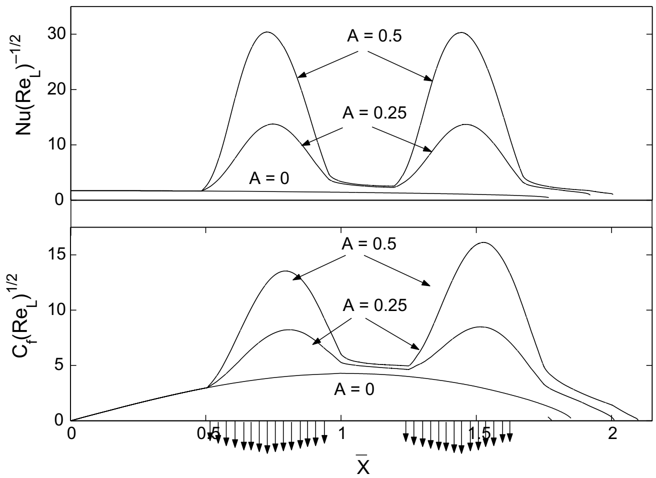

Fig. 3. Effect of suction ($A > 0$) on the skin friction and heat transfer coefficient for $T_\infty = 18.7$ °C, $\Delta T_w = 10$ °C, $Ec = 0.05$ and $w^* = 2\pi$. Slot locations at $\bar{x}_0 = 0.5, \bar{x}_1 = 1.25$.

shown in Fig. 3, it is observed that the point of separation moves upstream with an increase in the magnitude of the injection parameter ($A < 0$, see Fig. 4).

Fig. 5 displays the effect of slot locations on the point of separation. The solid line with A = 0 corresponds to the no mass transfer case, and the dashed line in Fig. 5 corresponds to the slots located at $\bar{x}_0 = 0.25$ and $\bar{x}_1 = 1$. The solid line with A = 0.5 represents slots positioned at $\bar{x}_0 = 0.5$ and $\bar{x}_1 = 1.25$, respectively. It is to be noted that the point of separation moves further downstream when the locations of the slots are moved downstream. Thus, the point of separation can be delayed by non-uniform double slot suction ($A > 0$) and also by positioning the slots further downstream.

5. Conclusions

The present study effectively compares the significance of non-uniform double slot suction/injection of laminar boundary layer flows over a cylinder. The numerical investigation shows that the point of separation can be delayed by non-uniform double slot suction if the mass transfer rate is increased and also if the slots are positioned further downstream. The results also indicate that the effect of non-uniform double slot injection is just the reverse. In addition, the investigation reveals that double slot suction is found to be more effective compared to single slot suction in delaying separation.

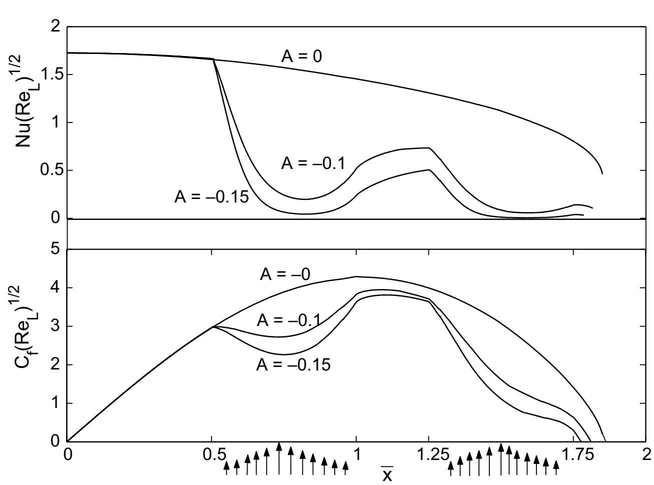

Fig. 4. Effect of injection ($A < 0$) on the skin friction and heat transfer coefficient for $T_\infty = 18.7$ °C, $\Delta T_w = 10$ °C, $Ec = 0.05$ and $w^* = 2\pi$. Slot locations at $\bar{x}_0 = 0.5, \bar{x}_1 = 1.25$.

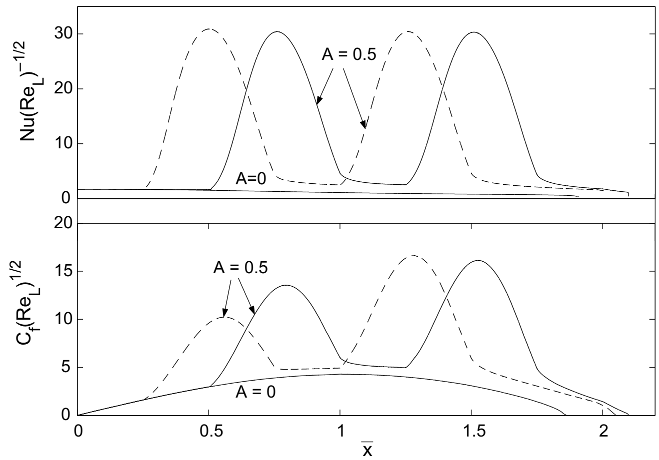

Fig. 5. Effect of slot locations $(\bar{x}_0, \bar{x}_1)$ on skin friction and heat transfer coefficients when $\omega^* = 2\pi$, $T_\infty = 18.7$, $T_w = 28.7$ and $Ec = 0.05$. (———) slot locations at $\bar{x}_0 = 0.5, \bar{x}_1 = 1.25$. (------) slot locations at $\bar{x}_0 = 0.25, \bar{x}_1 = 1.0$.

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